Multi-Agent Owner-Assisted Scoring Mechanisms

Shi Feng[1] (co-advised by Prof. Yuhao Wang[1] and Prof. Weijie Su[2])

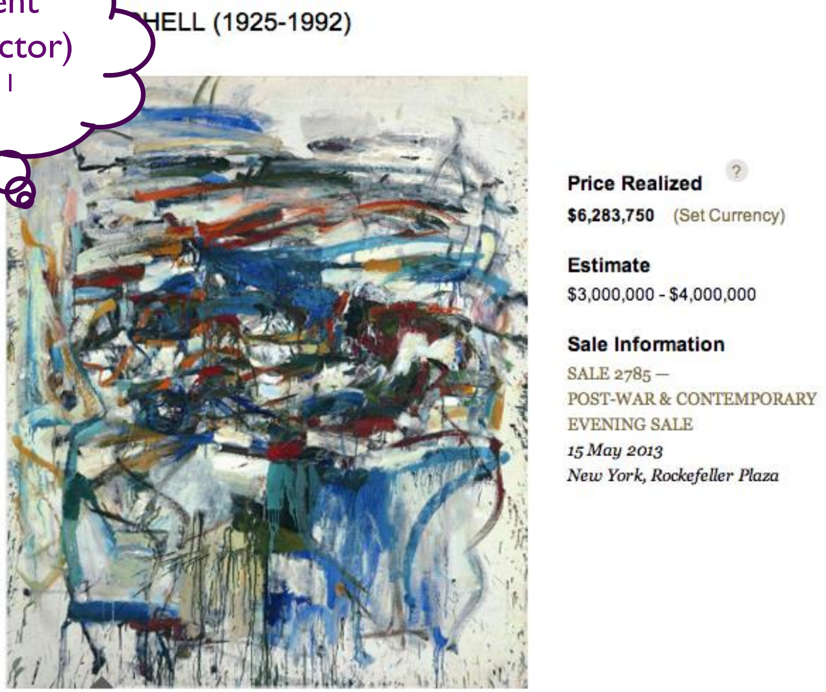
2022.12.18

1. INSTITUTE OF INTERDISCIPLINARY INFORMATION SCIENCES (IIIS), TSINGHUA UNIVERSITY
2. WHARTON STATISTICS AND DATA SCIENCE DEPARTMENT, UNIVERSITY OF PENNSYLVANIA

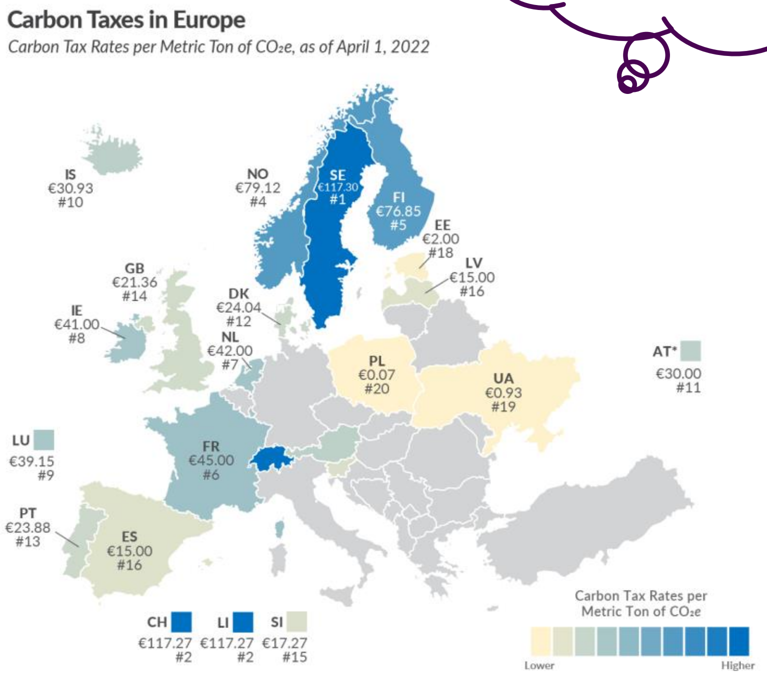
Contents

Why we need owner-assisted scoring?

Single-agent (artist/collector) scoring. [1]



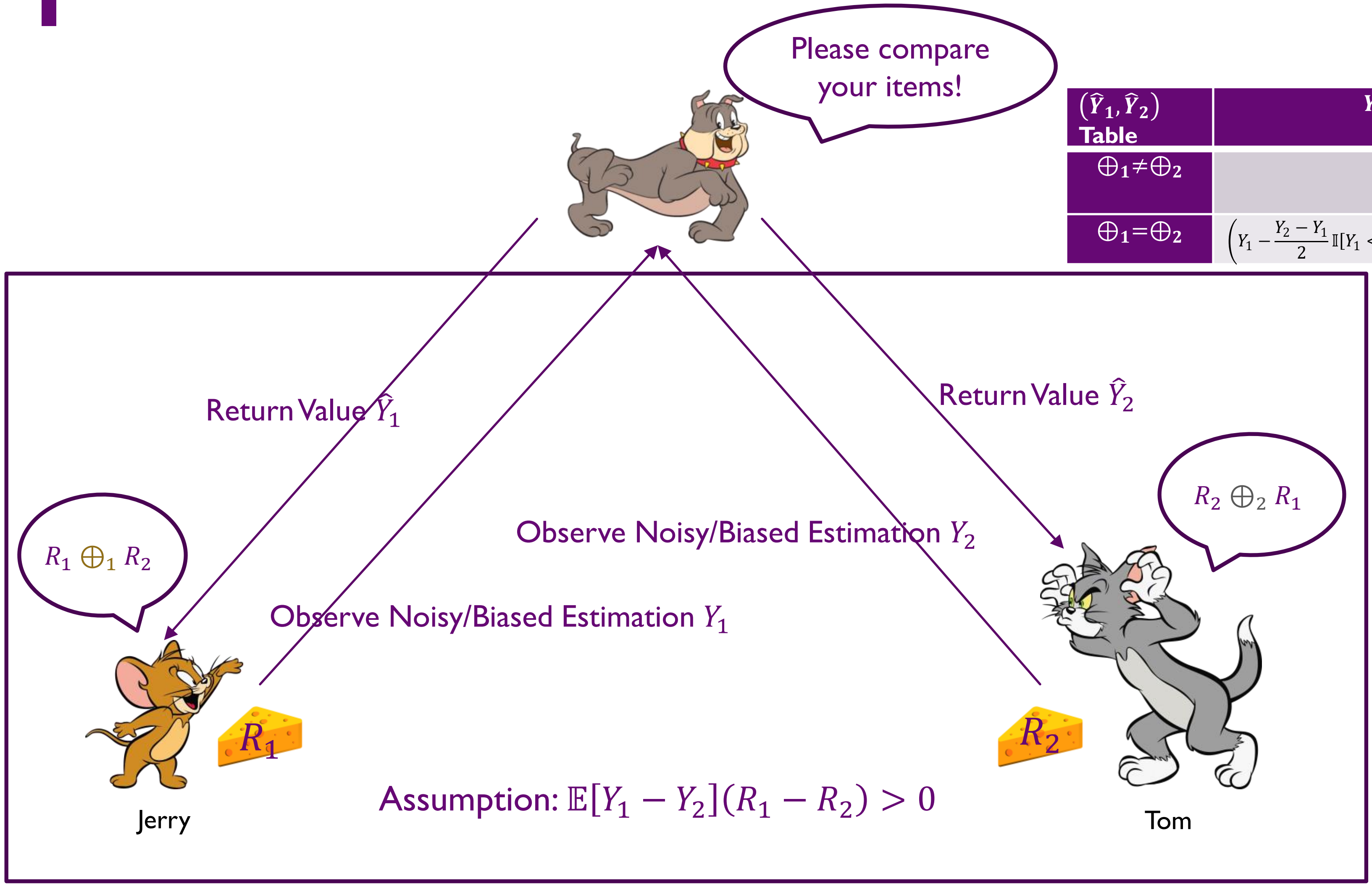
Taxing requires information from multiple owners.



Soliciting information from owners to improve value evaluation!

Contents

Mechanism $\mathcal{M}_2$ for Two Agents



Our Mechanism Design

$(\hat{Y}_1, \hat{Y}_2)$ Table	$Y_1 \oplus_1 Y_2$	$\neg(Y_1 \oplus_1 Y_2)$
$\oplus_1 \neq \oplus_2$	(Y_1, Y_2)	$\left(\frac{Y_1 + Y_2}{2}, \frac{Y_1 + Y_2}{2}\right)$
$\oplus_1 = \oplus_2$	$\left(Y_1 - \frac{Y_2 - Y_1}{2}\mathbb{I}[Y_1 < Y_2], Y_2 - \frac{Y_1 - Y_2}{2}\mathbb{I}[Y_1 > Y_2]\right)$	$\left(Y_1 - \frac{Y_2 - Y_1}{2}\mathbb{I}[Y_1 < Y_2], Y_2 - \frac{Y_1 - Y_2}{2}\mathbb{I}[Y_1 > Y_2]\right)$

Satisfying two design goals for $\hat{Y}_1, \hat{Y}_2$:

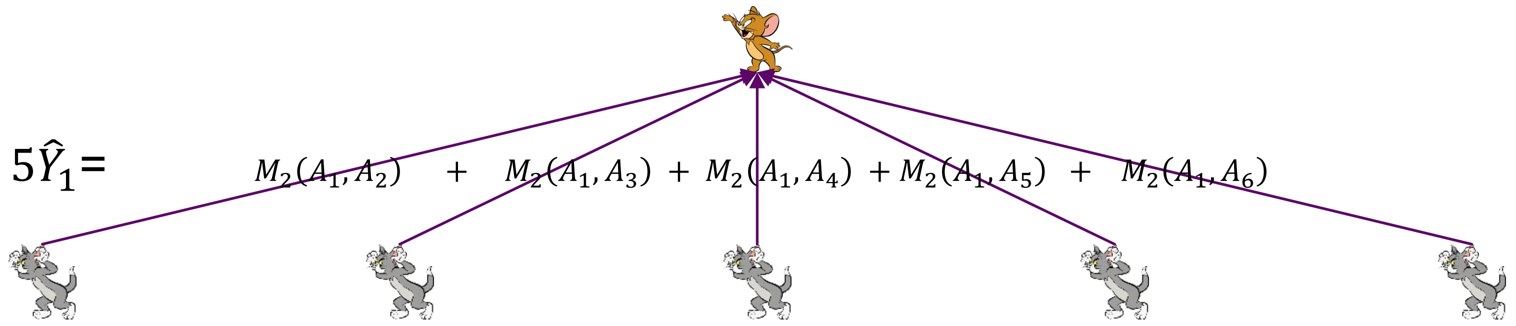
- incentive-compatibility (truthful $\oplus_1$ and $\oplus_2$),
- estimation improvement ($|\hat{\boldsymbol{Y}} - \boldsymbol{R}|_2 \leq |\boldsymbol{Y} - \boldsymbol{R}|_2$ if agents are both truthful).

Contents

Mechanism $\mathcal{M}_n$ for Multiple (n) Agents

Mechanism $\mathcal{M}_n$:

- Step 1: get noisy/biased estimation of $\mathbf{Y}$ for true item values $\boldsymbol{R}$.
- Step 2: for each pair $P_k = (A_i, A_j)$ of agents in pair set $\boldsymbol{P}$, perform mechanism $\mathcal{M}_2$ and get estimations $M_2(A_i, A_j)$ and $M_2(A_j, A_i)$ for those two agents, respectively.
- Step 3: return estimated value $\hat{Y}_i = \frac{\sum_{j=1, j\neq i}^{n} M_2(A_i, A_j)}{n-1}$ where $M_2(A_i, A_j)$ is set as $\frac{Y_i}{n-1}$ if (A_i, A_j) is not in $\boldsymbol{P}$.



$$5\hat{Y}_1 = M_2(A_1, A_2) + M_2(A_1, A_3) + M_2(A_1, A_4) + M_2(A_1, A_5) + M_2(A_1, A_6)$$

According to linearity of expectation, one could verify that $\mathcal{M}_n$ still satisfies **incentive-compatible** and **estimation improvement**.

Several Extensions

- We use the identity utility function (or linear utility function) previously. I also prove that with an assumption on $\boldsymbol{Y}$, our mechanisms $\mathcal{M}_2$ and $\mathcal{M}_n$ still guarantee **incentive-compatible** when agents have **bilipschitz utility functions**.

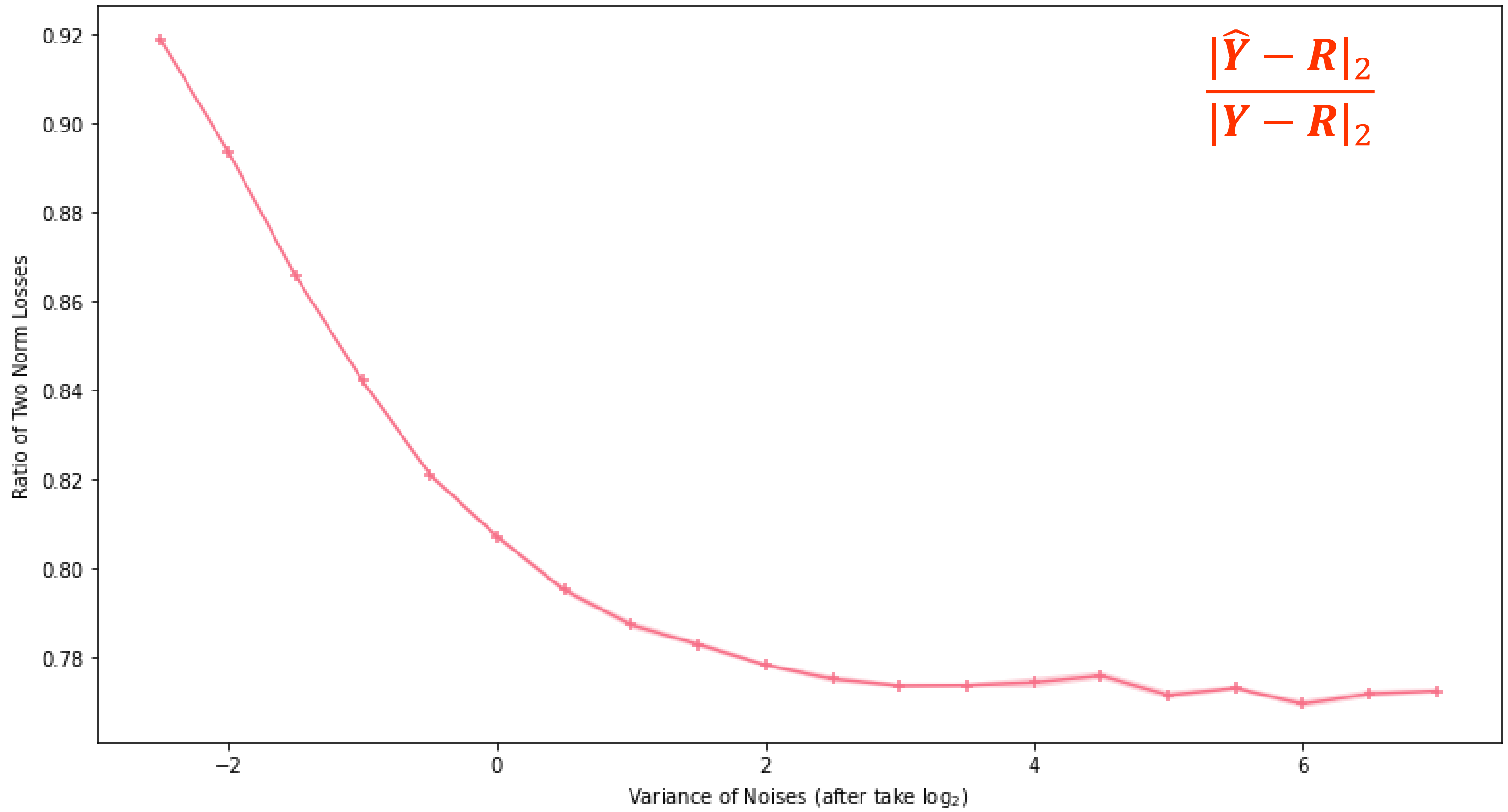
$$\kappa_1|x-y| \leq |U(x)-U(y)| \leq \kappa_2|x-y|$$

- By adding a constant ϵ smaller than 0.5, new mechanisms $\mathcal{M}_2'$ and $\mathcal{M}_n'$ support strictly **incentive-compatible** for multi-agent setting while each agent has **multiple items**.

$\hat{\boldsymbol{Y}}$ Design for $\mathcal{M}_2'$ and $\mathcal{M}_n'$

$(\hat{Y}_1, \hat{Y}_2)$ **Table**	$Y_1 \oplus_1 Y_2$	$\neg(Y_1 \oplus_1 Y_2)$
$\oplus_1 \neq \oplus_2$	(Y_1, Y_2)	$\left(\frac{Y_1+Y_2}{2} + \epsilon\frac{Y_1-Y_2}{2}\mathbb{I}[\oplus_1 = >] + \epsilon\frac{Y_2-Y_1}{2}\mathbb{I}[\oplus_1 = <], \frac{Y_1+Y_2}{2} + \epsilon\frac{Y_2-Y_1}{2}\mathbb{I}[\oplus_2 = >] + \epsilon\frac{Y_1-Y_2}{2}\mathbb{I}[\oplus_2 = <]\right)$
$\oplus_1 = \oplus_2$	$\left(Y_1 - (1+\epsilon)\frac{Y_2-Y_1}{2}\mathbb{I}[Y_1 < Y_2], Y_2 - (1+\epsilon)\frac{Y_1-Y_2}{2}\mathbb{I}[Y_1 > Y_2]\right)$	$\left(Y_1 - (1+\epsilon)\frac{Y_2-Y_1}{2}\mathbb{I}[Y_1 < Y_2], Y_2 - (1+\epsilon)\frac{Y_1-Y_2}{2}\mathbb{I}[Y_1 > Y_2]\right)$

Contents



Simulations

We run $\mathcal{M}_n$ for all pairs of agents on random item values $\boldsymbol{R}$ and standard distribution noises $\boldsymbol{\epsilon}$ such that $\boldsymbol{Y} = \boldsymbol{R} + \boldsymbol{\epsilon}$. $\widehat{\boldsymbol{Y}}$ generated by $\mathcal{M}_n$ achieves **8%-20% improvement** over $\boldsymbol{Y}$.

Q&A

Thank you for your attention!